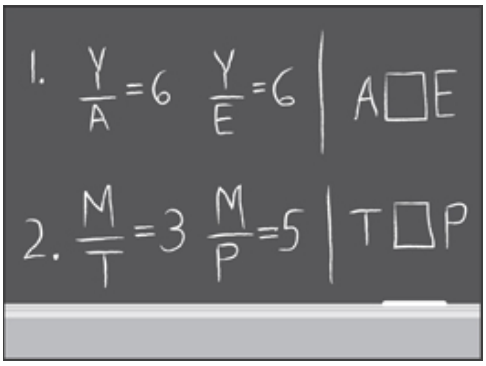

Introducing letters to represent quantities, as in these first-grade math problems, helps students prepare for algebra. (Source: Curriculum Research and Development Group, Univ. of Hawaii)

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The Algebra Problem

How to elicit algebraic thinking in students before eighth grade

by LAURA PAPPANO

It's Crazy Hair Day at Marshall Elementary School in Boston's Dorchester neighborhood—which is perfect, because Tufts University researcher Bárbara Brizuela has brought a hat.

In the stovepipe style and made from oaktag paper, the hat is one foot tall. Brizuela then asks, "If I'm five and a half feet tall, how tall will I be with the hat on?" Second-grader Jasmine, smiley in a pink sweatsuit, answers, "Six and a half feet." Rather than say, "Right!" Brizuela offers another question: "How do you know?"

Thus begins a math conversation that researchers like Brizuela believe may hold the key to tackling one of our biggest school bugaboos: algebra. As they talk, Jasmine uses words, bar graphs, and a table to describe how tall each person they discuss will be if they put on the hat. Jasmine creates a rule—"add one foot to the number you already had"—and applies it to an imaginary person 100 feet tall.

Brizuela even throws out a variable. "So, to show someone whose height I don't know, I will use z feet," she says, adding a z to Jasmine's table. "What should I do now?" Jasmine pauses. "This is kind of hard," she says. Brizuela, whose pilot study explores mathematical thinking among children in grades K–2, understands. "Would you like to use a different letter?" she asks, erasing the z and replacing it with a y. Jasmine smiles. She picks up her pencil and easily jots down the rule: y + 1 = z feet.

A Dreaded, Scary Subject

It may seem adorable that young children are stumped if asked to add 1 to z but not if asked to add 1 to y, but to Brizuela, director of the Mathematics, Science, Technology, and Engineering Education Program in Tuft's education department, it reveals the reasoning capacity of young minds and the need to engage them in algebraic thinking long before it becomes a dreaded and scary subject.

To many, algebra is about the first or last three letters of the alphabet, and it provokes groaning, trash talk (think Forever 21's "Allergic to Algebra" T-shirt), and heated debate. Should it be mandated? At what grade? Algebra's status as a "gatekeeper course" has made it a touchstone on matters of access and equity. As a result, in many places it's become a graduation requirement.

Back in the early 1980s, one-quarter of high school graduates never even took algebra, says Daniel Chazen, director of the Center for Mathematics Education at the University of Maryland. Today, educators are pushing students to take algebra even before high school. According to the National Assessment of Educational Progress (NAEP), the number of students taking Algebra I in eighth grade more than doubled between 1986 and 2011, from 16 to 34 percent. Strikingly, eighth-grade NAEP math test scores have edged up too, with 43 percent scoring advanced or proficient in 2011, compared with 27 percent in 1996.

But amid the good news is a troubling reality: Many kids are failing algebra. In California, where standards call for Algebra I in grade 8, a 2011 EdSource report shows that nearly one-third of those who took the course—or 80,000 students—scored "below basic" or "far below basic." In districts across the country, failure rates for Algebra I vary but run as high as 40 or 50 percent, raising questions about how students are prepared—and how the subject is taught.

Starting Algebra Early

Why is algebra so hard? For many students, math experts say, it is a dramatic leap to go from the concrete world of computation-focused grade school math to the abstract world of algebra, which requires work with variables and changing quantitative relationships. It is not just the shock of seeing letters where numbers have been but also the type of thinking those letters represent.

"In arithmetic, you are dealing with explicit numbers," says Hung-Hsi Wu, a professor emeritus of mathematics at the University of California, Berkeley. "Algebra says, 'I have a number; I don't know what it is, but three times it and subtract three is 15.' You have a

number floating out there, and you have to catch it. It is the thinking behind catching the number that baffles students."

While some argue that children must be developmentally ready to learn algebra—around ages 11–13, when they can grasp abstract thought—Brizuela and others say it's critical to introduce it earlier. "Kids need to develop some comfort with these tools," she says. "Babies are exposed to written and spoken language, and after six years we expect them to become somewhat fluent with that. In math, we just drop it on them like a bomb."

Brizuela's research spans more than a dozen years and seeks to find out if explicitly teaching algebraic thinking, including a comfort with letter variables and the ability to express mathematical values in multiple forms (Jasmine's words, table, and bar graph), might be helpful later on.

In a study to be published in October in *Recherches en Didactique des Mathématiques*, a French math education journal, Brizuela and her colleagues tracked 19 students in Boston Public Schools in grades 3, 4, and 5 who received weekly algebra lessons plus homework, as compared with a control group, and followed them through middle school. Results showed that those students outperformed their peers on algebra assessments given in grades 5, 7, and 8 and drawn from NAEP, Massachusetts state tests, and the Trends in International Mathematics and Science Study, or TIMMS.

Building Math Minds

Central to Brizuela's work is a striking idea: Rather than pushing eighth-grade or high school algebra down to elementary school, she begins with what children already tend to do, such as generalizing. For example, when children hear the word "hundred," they know to add two zeros. Brizuela uses that natural ability to lure children into thinking about quantitative relationships that then become algebraic rules. This exercises their natural mathematical reasoning, which is often pushed aside in favor of getting the "right" answer or learning to memorize or compute (see sidebar "Laying the Groundwork for Algebra"). **Laying the Groundwork for Algebra**

Here are three things that teachers can do to encourage algebraic thinking, according to researchers:

• **Broaden your definition of the equal sign**. Children should be trained to view an equal sign (=) as balancing an equation, not as a command to produce an answer, says Cathy L. Seeley, a senior fellow at the Charles A. Dana Center at the University of Texas at Austin. "If you help them be fluid with what the equal sign is, it starts helping children to grasp algebra."
• **Introduce letters, carefully**. Including letters in math problems early on can help children grow comfortable with seeing and working with them, but they can also be misleading. Some young children can correlate a letter with its order in the alphabet, like a (first) or z (last). Tufts University researcher Bárbara Brizuela does not use x as a variable because children view it as "crossing out."
• **Talk about math**. So much of grade school math is "what you do with paper," but paper work is typically about computation and answers, not mathematical reasoning, says former math teacher Paul Goldenberg of the Educational Development Center in Waltham, Mass. Presenting problems orally and framing them as a continuation of earlier ideas, rather than a "frightening new language," can help, he says.

Similarly, Barbara J. Dougherty, Richard G. Miller Chair of Mathematics Education at the University of Missouri, observes that first-graders naturally compare, often to be sure they have the same amount (of whatever is in question) as somebody else.

"In starting with children at six, rather than starting with numbers, we ask, 'How do you know if you have more than somebody else or less?'" says Dougherty. She and her colleagues use measurement as a vehicle for discussing comparisons of, say, the height of a cereal box to the length of a pencil. Then, instead of writing down "the height of the cereal box" and "the length of the pencil," she says, "we'll say, 'Let b represent the height of the cereal box and l be the length of the pencil.' It sounds pretty simple, but it is actually pretty powerful." Dougherty, who has been following a cohort of students at the University Laboratory School in Honolulu, Hawaii, since 2001, says that by the time the students reach high school, they consistently outperform peers in their understanding of algebraic concepts like variables and quantitative relationships.

In the Lab School, whose student population reflects the state's socioeconomic and racial composition, first-grade teacher Maria DaSilva says that rather than presenting the students with, say, a number line right off, she lets the class puzzle through a problem—sometimes over the course of days—until they realize that having a number line will help them in their work (see sidebar "Algebra in First Grade?"). **Algebra in First Grade?**

At the University Laboratory School in Honolulu, Hawaii, first-graders solve algebraic problems disguised as real-life dilemmas. One such problem involves figuring out how much growth hormone a doctor must give a population of shrimp for them to reach a certain size, given that over time they need different amounts because previous doses have made them grow. When the doctor "gets confused" about how much growth hormone to give, the children must find a way to keep track.

Teacher Maria DaSilva has students measure out liquid "doses" to "feed" the growing shrimp by marking on masking tape placed along the side of a container. Later, she removes the tape and places it horizontally on a piece of paper to become a number line. This exercise gets students thinking about changing variables as opposed to fixed amounts and demonstrates that between whole units there exist partial units—or fractions—which experts say is absolutely critical to understanding and solving algebraic equations. A common reason students get tripped up in algebra is that they don't understand what fractions really represent and how to manipulate them, experts say.

The Teaching Challenge
The drive to improve U.S. math performance among students has focused on two main worries: (1) Are students well enough prepared, and (2) are teachers prepared enough to teach math well?

William Schmidt, professor and codirector of the Education Policy Center at Michigan State University, says the new Common Core standards likely to be adopted by most states for 2013–2014, "capture the logic of mathematics,"—an upgrade from the seemingly unrelated lessons that have made learning math "like reading the phone book."

But he wonders: Will teachers be able to teach it? In a 2010 study, *Breaking the Cycle: An International Comparison of U.S. Mathematics Teacher Preparation*, comparing U.S. primary and middle school teachers with peers in 16 countries, Schmidt and his colleagues found that American teachers had "weak training mathematically" and less math coursework than teachers in high-performing nations. "We have this new demanding curriculum in the middle grades and teachers who are ill prepared to teach it," he warns.

Meanwhile, excitement over raised standards has been met with a worry: What about the kids who are struggling now? Math researchers, like James J. Lynn at the University of Illinois at Chicago, with colleagues in New York and Seattle, are in the third year of a four-year National Science Foundation–funded project to study 17,000 high school students who struggle with algebra. Their approach is to promote sense-making, which they say has been lacking in many students' earlier algebra experiences.

Along with work aimed at bolstering students' sense of how quantities relate—including filling deficits as they go rather than undertaking long periods of "re-teaching"—the project also seeks to change the mindset around algebra. Instead of viewing algebra as insurmountable, students learn that applying effort and wrestling with problems can grow brain connections and make them smarter and better at math. "We try to shape their attitudes of themselves as capable learners," says Lynn. The program is showing some gain, with about half the students scoring "high mastery" after the course (most students scored "low mastery" prior to the course).

Given such difficulty, one has to wonder: Why even learn algebra?

According to Jon R. Star, associate professor at the Harvard Graduate School of Education, that's like asking: "Why are they reading *Wuthering Heights*?" Star says the answer is that—like literature—algebra tells us something about human nature and understanding. Algebra, he says, "is our students' first exposure to what mathematics is." It offers students the sort of critical thinking about mathematical ideas that simply doesn't come with the computation skills of early school math. Instead, he argues, we should simply point out that, when we get to algebra, "we are here to learn some mathematics." Not computation. Not calculation. But real math.

Freelance education writer and author Laura Pappano is a frequent contributor to the Harvard Education Letter.

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For Further Information

For Further Information

J. Babcock et al. *Breaking the Cycle: An International Comparison of U.S. Mathematics Teacher Preparation*. East Lansing, MI: Center for Research in Mathematics and Science Education, 2010.

D. Chazan. “The Shifting Landscape of School Algebra in the United States: No Child Left Behind, High School Graduation Requirements, Principles and Standards, and Technology.” In *Algebra and Algebraic Thinking in School Mathematics*, edited by C. Greenes and R. Rubenstein, 19–33. Reston, VA: National Council of Teachers of Mathematics, 2008.

J. R. Star and B. Rittle-Johnson. “Making Algebra Work: Instructional Strategies That Deepen Student Understanding, Within and Between Algebraic Representations.” *ERS Spectrum 27*, no. 2 (2009), 1–18.

T. Williams et al. *Improving Middle Grades Math Performance: A Closer Look at District and School Policies and Practices, Course Placements, and Student Outcomes in California*. Mountain View, CA: EdSource, 2011.